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Gauged M-flation after BICEP2

A. Ashoorioon [a], M.M. Sheikh-Jabbari [b,c,*]

[a] *Consortium for Fundamental Physics, Physics Department, Lancaster University, LA1 4YB, United Kingdom*
[b] *School of Physics, Institute for Research in Fundamental Sciences (IPM), P.O. Box 19395-5531, Tehran, Iran*
[c] *Department of Physics, Kyung Hee University, Seoul 130-701, Republic of Korea*

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A B S T R A C T

In view of the recent BICEP2 results [arXiv:1403.3985] which may be attributed to the observation of B-modes polarization of the CMB with tensor-to-scalar ratio $r = 0.2^{+0.07}_{-0.05}$, we revisit M-flation model. Gauged M-flation is a string theory motivated inflation model with Matrix valued scalar inflaton fields in the adjoint representation of a $U(N)$ Yang–Mills theory. In continuation of our previous works, we show that for a class of M-flation models the action for these inflaton fields can be such that the "effective inflaton field" ϕ has a double-well Higgs-like potential, with minima at $\phi = 0, \mu$. We focus on the $\phi > \mu$, symmetry-breaking region. We thoroughly examine predictions of the model for r in the 2σ region allowed for n_S by the Planck experiment. As computed in [arXiv:0903.1481], for $N_e = 60$ and $n_S = 0.96$ we find $r \simeq 0.2$, which sits in the sweet spot of BICEP2 region for r. We find that with increasing μ arbitrarily, n_S cannot go beyond $\simeq 0.9670$, the scalar spectral index for the quadratic chaotic potential. As n_S varies in the 2σ range which is allowed by Planck and could be reached by the model, r varies in the range $[0.13, 0.26]$. Future cosmological experiments, like the CMBPOL, that confines n_S with $\sigma(n_S) = 0.0029$ can constrain the model further. Also, in this region of potential, for $n_S = 0.9603$, we find that the largest isocurvature mode, which is uncorrelated with curvature perturbations, has a power spectrum with the amplitude of order 10^{-11} at the end of inflation. We also discuss the range of predictions of r in the hilltop region, $\phi < \mu$.

1. Introduction

Inflation, and in particular slow-roll inflation, has emerged as the leading framework to understand and explain the recent cosmological CMB observations, most notably by PLANCK mission [1] and BICEP2 [2]. The most common models of inflation are those involving one or more scalars with canonical kinetic terms and a (carefully) chosen or designed potential (minimally) coupled to Einstein gravity. Despite the theoretical simplicity and addressing some theoretical issues in Big Bang early Universe cosmology, the wealth of the new observational data has made it increasingly difficult to find inflationary models which do well with data, as well as being free of theoretical issues, like naturalness, stability of inflaton potential and having a natural appearance and embedding into high energy physics models, such as beyond SM particle physics models or string theory.

Power spectrum of tensor modes, within the Einstein gravity theory, is independent of the details of the inflaton sector and sets the value of Hubble scale during inflation H. The recent observations of BICEP2 if attributed to the primordial B-mode polarization in the CMB power spectra,[1] within the inflationary setup is generically related to the tensor modes and set $H \simeq 3\text{–}4 \times 10^{-5} M_{\rm pl}$ [3], where $M_{\rm pl} \equiv (8\pi G_N)^{-1/2} = 2.43 \times 10^{18}$ GeV is the reduced Planck mass. The ratio of power spectra of tensor and scalar modes in the inflationary models with canonical kinetic terms is $r = 16\epsilon$

[1] As was stressed in [4] and [5], the BICEP data analysis may have underestimated the contribution of combination of Galactic foregrounds and lensed E-modes. Including such foreground effects, within the current uncertainties, $r = 0$ is also compatible with the BICEP CMB data. The first version of this paper appeared before such scrutiny in the foreground effects and is relevant if the value of r at $\ell = 80$ obtained after "realistic" foreground removal turns out to be larger than the corresponding value of r at the Planck experiment pivot scale. Planck has recently put out its data for the power spectrum of polarized dust emission [6] in which they have extrapolated the 353 GHz Planck data to 150 GHz of BICEP2 frequency range. BICEP2 result is still within the uncertainties and thus such studies are not yet conclusive.

* Corresponding author.
E-mail addresses: a.ashoorioon@lancaster.ac.uk (A. Ashoorioon), jabbari@theory.ipm.ac.ir (M.M. Sheikh-Jabbari).

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and the spectral tilt n_S which parameterizes the scale dependence of CMB power spectrum, is $n_S - 1 = -6\epsilon + 2\eta$, where ϵ and η are the slow-roll parameters. Currently available data indicate that $\epsilon \sim \eta \sim 0.01$.

The simplest inflationary models which can produce such a relatively large value $r \sim 0.2$ [2] are slow-roll large field models [7], and the most appealing model in this family is the $\frac{1}{2}m^2\phi^2$ chaotic model [8] with $m \simeq 6 \times 10^{-6} M_{\rm pl}$. To produce these values of r we generically need super-Planckian field roaming, $\Delta\phi \sim 10 M_{\rm pl}$, crystallized in the Lyth bound [9]. The Lyth bound and $r \sim 0.2$ appear as a challenge to supergravity and string theory motivated inflationary models in which $\phi/M_{\rm pl}^2$ has generically a geometric meaning and is associated with some physical length [10] which consistency of the theory requires it to be sub-Planckian.

Moreover, recent data together with number of e-folds N_e required for solving cosmological flatness and horizon problems, $N_e \sim 50$–60, has other challenging implications for the inflaton potential in slow-roll large field or string theory motivated models: dimensionful parameters of the inflaton potential should be hierarchically smaller than H and dimensionless parameters much smaller than one. That is, we are in unnatural regions of the parameters space of these models and that we need to make sure the classical as well as quantum stability of the shape of the potential. The η-problem [10] in string or supergravity motivated models alludes to this problem.

To ameliorate the above mentioned issues, two general ideas have been proposed: steepness of the potential and its shape may be fixed by imposing symmetries. (One cannot rely on supersymmetry, due to the fact that on an inflationary background supersymmetry is spontaneously broken at scale H.) The super-Planckian fields have been remedied by employing large number of fields which are abundant in supergravity or string theory setups all assisting to run forward the inflation [11].[2] Using large number of fields of almost equivalent mass, as is done in [14], however, has its own drawbacks. Having N number of degenerate fields will effectively reduce the cutoff scale where quantum gravity effects are expected to set in (which otherwise may be taken to be $M_{\rm pl}$) is also reduced to $M_{\rm pl}/\sqrt{N}$ [15,16]. This will bring back the problem of super-cutoff roaming and quantum instability. Moreover, for large N, $\sim 10^9$, light fields this will also lead to dominance of quantum field perturbations over the classical inflaton rolling in N-flation, and hence causing a never-ending eternal inflation [17]. There are nonetheless multi-axion models that avoid these problems [18].

In [19] we introduced Matrix inflation, or M-flation, which despite being a string theory motivated model conveniently bypasses the above issues. M-flation action may be viewed as the low energy theory of a stack of N D3-branes probing certain type IIb supergravity background with RR three-form fluxes.[3] This model, is hence as a $U(N)$ supersymmetric Yang–Mills theory perturbed by a mass term for three of its scalar fields which are in $N \times N$ adjoint representation of $U(N)$, as well as a cubic term for these scalars induced from the background RR-form. Although we are dealing with $5N^2$ gauge and scalar field modes, one can effectively reduce the theory to a single field sector with a double well Higgs-like inflaton potential. Of course, this theory contains a plethora/landscape of models which effectively behave as multi-field models [20]. In this work we focus on the effective single field model. M-flation potential is protected by the supersymmetry of the bulk [19]. It does not suffer from super-cutoff field roaming and falling into the eternal inflation phase because the spectrum of its fields around the effective single field model is not degenerate and all the modes around the classical inflationary path (which constitute $5N^2 - 1$ modes in gauged M-flation) have a hierarchical mass spectrum [16]. This hierarchical mass of isocurvature modes is the distinctive feature of M-flation compared to other multi-filed models like N-flation. This mass spectrum is such that only a small number of the fields N_s, $N_s \ll N^2$, have masses below the effective cutoff scale. These are the modes contributing to the effective reduced cutoff, which is hence $M_{\rm pl}/\sqrt{N_s}$ [16]. The reduced cutoff in (gauged) M-flation will also help to avoid the η-problem that can be generated at one loop level from the interactions of the inflaton with the graviton [21]. In addition, as we will see, M-flation, does not involve hierarchically small dimensionful parameters and its dimensionless parameters are of order one.

Therefore, M-flation is a very theoretically appealing model. In this short note, we hence update our analysis of this model and show how it complies with the recent data and in particular the BICEP2. We show that unlike most of the string theory motivated inflation models, it can produce large enough tensor-to-scalar ratios as required by the data. The rest of this work is organized as follows. In Section 2, we review the setup of gauged M-flation and some basic analysis. In Section 3, we confront gauged M-flation with the current observational data. In the last section we summarize our results and make concluding remarks.

2. Quick review of gauged M-flation

Gauged M(atrix)-flation [19,20,16] is a sector of deformation of $\mathcal{N} = 4$ $U(N)$ supersymmetric Yang–Mills theory coupled to 4d Einstein gravity described by the action

$$S = \int d^4x\sqrt{-g}\left(\frac{-M_P^2}{2}R - \frac{1}{4}\mathrm{Tr}\big(F_{\mu\nu}F^{\mu\nu}\big) - \frac{1}{2}\sum_{i=1}^{3}\mathrm{Tr}\big(D_\mu\Phi_i D^\mu\Phi_i\big) - V\big(\Phi_i, [\Phi_i, \Phi_j]\big)\right), \tag{2.1}$$

where D_μ is the gauge covariant derivative, $F_{\mu\nu}$ is the gauge field strength, and Φ_i, $i = 1, 2, 3$, are three $N \times N$ hermitian scalars in the adjoint representation of $U(N)$ gauge symmetry:

$$D_\mu\Phi_i = \partial_\mu\Phi_i + ig_{\rm YM}[A_\mu, \Phi_i],$$
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig_{\rm YM}[A_\mu, A_\nu], \tag{2.2}$$

and the Tr is over $N \times N$ matrices. This action which can be realized in string theory as the low energy theory of N D3-branes probing certain 10b IIB plane-wave background with RR three-form field background [19], if the potential term $V(\Phi_i, [\Phi_i, \Phi_j])$ has the following form

$$V = \mathrm{Tr}\left(-\frac{\lambda}{4}[\Phi_i, \Phi_j][\Phi_i, \Phi_j] + \frac{i\kappa}{3}\epsilon_{jkl}[\Phi_k, \Phi_l]\Phi_j + \frac{m^2}{2}\Phi_i^2\right), \tag{2.3}$$

with

[2] Besides the M-flation, which is the focus of this work, among string theory motivated models monodromy inflation [12] is seemingly dealing better in resolving the super-Planckian field roaming and, the steepness of the potential and η-problem by using axions as inflaton field; super-Planckian field roaming is resolved noting the periodic nature of axion field and the shape of potential is protected by the (approximate) shift symmetry of the axion potential. The original model fails to comply with the BICEP data due to small r it predicts. Some variations like [13] are in a better shape with regard to compatibility with the data.

[3] Despite these appealing features, M-flation has not yet been shown to be a fully-fledge string theory model. The main open issue here has to do with getting 4d gravity on the stack of D3-branes. For the latter one needs to have a consistent (CY_3) compactifications with moduli fixed and also one needs to makes sure that the backreaction of D3-branes on the compactification are negligible, or under control. We hope to address this issue in our later publications.

$$\lambda = 8\pi g_s = 2g_{\rm YM}^2, \qquad \lambda m^2 = 4\kappa^2/9, \tag{2.4}$$

where $i = 1, 2, 3$ and g_s is the string coupling constant. It is notable that in terms of coordinates of stack of N D3-branes X^i

$$\Phi_i \equiv \frac{X_i}{\sqrt{(2\pi)^3 g_s l_s^2}}, \tag{2.5}$$

where ℓ_s is the string length.

2.1. Equations of motion for the scalar and gauge fields

$$\begin{aligned} &D_\mu D^\mu \Phi_i + \lambda\big[\Phi_j, [\Phi_i, \Phi_j]\big] - i\kappa\epsilon_{ijk}[\Phi_j, \Phi_k] - m^2\Phi_i = 0, \\ &D_\mu F^{\mu\nu} - ig_{\rm YM}\big[\Phi_i, D^\nu \Phi_i\big] = 0. \end{aligned} \tag{2.6}$$

2.2. Reduction to effective single-field sector

As noted in [16,19,20], one can consistently restrict the classical dynamics to the $SU(2)$ sector, where one effectively deals with a single scalar field $\hat{\phi}$ and the matrices Φ_i are

$$\Phi_i = \hat{\phi}(t) J_i, \quad i = 1, 2, 3, \tag{2.7}$$

where J_i are the N-dimensional irreducible representation of the $SU(2)$ algebra

$$[J_i, J_j] = i\epsilon_{ijk} J_k, \qquad \mathrm{Tr}(J_i J_j) = \frac{N}{12}(N^2-1)\delta_{ij}. \tag{2.8}$$

Hermiticity of Φ_i's and J_i's guarantees that $\hat{\phi}$ is a real scalar field. One can consistently turn off the gauge fields, *i.e.* $A_\mu = 0$, in the background as the $[\Phi_i, D_\nu\Phi_i]$ term in the equation of motion of the gauge field for the ansatz (2.7) is proportional to $[J_i, [J_i, A_\nu]]$. Therefore, the classical inflationary trajectory takes place in the $SU(2)$ sector of the scalar fields Φ_i's.

Plugging the ansatz into the action (2.1) and adding the four-dimensional Einstein gravity, one obtains

$$S = \int d^4x\sqrt{-g}\bigg[-\frac{M_P^2}{2}R + \bigg(-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V_0(\phi)\bigg)\bigg], \tag{2.9}$$

where

$$\begin{aligned} &\hat{\phi} = \big(\mathrm{Tr}\, J^2\big)^{-1/2}\phi = \bigg[\frac{N}{4}\big(N^2-1\big)\bigg]^{-1/2}\phi, \\ &V_0(\phi) = \frac{\lambda_{\rm eff}}{4}\phi^4 - \frac{2\kappa_{\rm eff}}{3}\phi^3 + \frac{m^2}{2}\phi^2, \end{aligned} \tag{2.10}$$

with

$$\begin{aligned} &\lambda_{\rm eff} = \frac{2\lambda}{\mathrm{Tr}\, J^2} = \frac{8\lambda}{N(N^2-1)}, \\ &\kappa_{\rm eff} = \frac{\kappa}{\sqrt{\mathrm{Tr}\, J^2}} = \frac{2\kappa}{\sqrt{N(N^2-1)}}. \end{aligned} \tag{2.11}$$

As we will see, assuming that the parameters, λ, κ and m^2, are related as in (2.4) the potential takes a "symmetry breaking" Higgs potential. With super-Planckian v.e.v.'s for the effective inflaton ϕ, the potential is capable of realizing inflation in different regions. We note that this may happen while the "physical field Φ and/or stringy lengths X^i are sub-Planckian or sub-stringy. To match the amplitude of density perturbations of order the Planck normalization, $P_S \simeq 2\times 10^{-9}$, $\lambda_{\rm eff}$ should be around 10^{-14}–10^{-13}. Assuming that the undressed quartic coupling, λ is of order one, one needs around few $\times\, 10^4$ D3 branes which is a reasonable number noting recent advances in flux compactification with large amount of flux. For such number of D3 branes, the individual physical displacement of the fields is about $10^{-6}M_{\rm pl}$.

2.3. Spectrum of gauged M-flation "spectator" modes

In the gauged M-flation, starting with three N^2 scalar fields Φ_i and four N^2 gauge fields we have $7N^2$ degrees of freedom. Picking up the $SU(2)$ sector (2.7), we have turned on only one configuration of these fields. The gauge fields satisfy the gauge symmetry of the action and equations of motion. Therefore we expect only $2N^2$ of the them remain as physical and, aside from the mode along the $SU(2)$ sector, in total we have $5N^2-1$ modes, which even though are classically frozen, could be excited quantum mechanically. One can show that the backreaction of these isocurvature modes on the inflationary background during slow-roll period, is very small [19,20,16]. Thus they are truly the "*spectator*" modes. Below we will present the spectrum of spectator modes.

- *Scalar modes*: The spectrum of scalar fluctuations could be obtained perturbing Φ_i's around the $SU(2)$ sector

$$\Phi_i = \hat{\phi} J_i + \Psi_i, \tag{2.12}$$

expanding the action to second order in Ψ's and diagonalizing the second order action. One then find two class of modes [22]:

  - α_j-modes: $\omega = -(j+2)$ and $0 \le j \le N-2$. Degeneracy of each α_j mode is $2j+1$. Therefore there are $(N-1)^2$ of these modes. The mass of these modes are

$$\begin{aligned} M_{\alpha_j}^2 = &\frac{1}{2}\lambda_{\rm eff}\phi^2(j+2)(j+3) \\ &- 2\kappa_{\rm eff}\phi(j+2) + m^2. \end{aligned} \tag{2.13}$$

The $\alpha_{j=0}$ mode is the quantum fluctuations of the direction along the $SU(2)$ sector which are nothing other than the adiabatic mode. Thus we have $(N-1)^2-1$ α-mode.

  - β_j-modes: $\omega = j-1$ and $1 \le j \le N$. Degeneracy of each β_j-mode is $2j+1$. Therefore there are $(N+1)^2-1$ of β-modes. Mass of β_j mode is

$$\begin{aligned} M_{\beta_j}^2 = &\frac{1}{2}\lambda_{\rm eff}\phi^2(j-1)(j-2) \\ &+ 2\kappa_{\rm eff}\phi(j-1) + m^2. \end{aligned} \tag{2.14}$$

- *Gauge modes*: Likewise for the gauge fields, turning on the A_μ's and expanding the action (2.1) to second order in them assuming that the scalar fields are in the $SU(2)$ sector, *i.e.* $\Phi_i = \hat{\phi} J_i$, one can obtain the mass spectrum of gauge modes solving the related eigenvalue problem [16]. The masses turn out to be

$$M_{A,j}^2 = \frac{\lambda_{\rm eff}}{4}\phi^2 j(j+1), \tag{2.15}$$

where $j = 0, \ldots, N-1$ and degeneracy of each mode is $2j+1$. $j = 0$ mode is massless and corresponds to the $U(1)$ sector in the $U(N)$ matrices. Degeneracy of each j gauge mode is $3(2j+1)$ for $j \ge 1$ modes and is two for $j = 0$ mode. Thus we have $3N^2-1$ vector field modes.
We note that there are N^2-1 "zero-modes" in the scalar sector, which are eaten up through the usual spontaneous breaking occurring due to expansion around the $SU(2)$ vacuum [22,16].

In summary we have $(N-1)^2-1$ α-modes, N^2+2N β-modes and $3N^2-1$ vector field modes, which altogether yields $5N^2-1$ modes, exactly the number of spectator (isocurvature) modes that we started with.

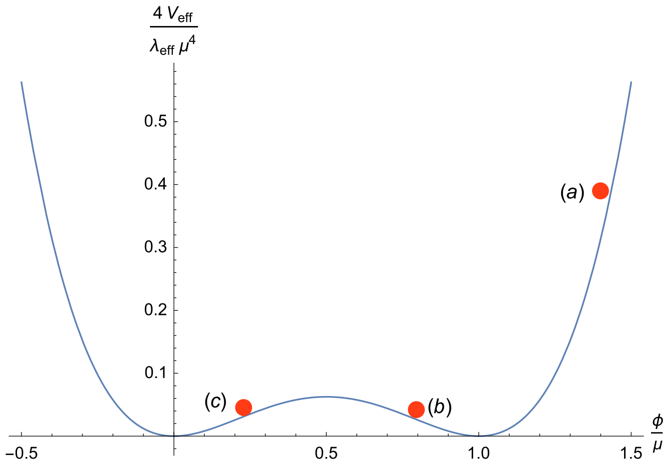

Fig. 1. The double-well potential which is derived in the context of M-flation. Confining to the $\phi \geq 0$ region, there are three regions one can use to inflate upon. Due to the symmetry of the potential the predictions of the model in regions (b) and (c) are the same in the (n_S, r) plane. This degeneracy breaks down at the level of isocurvature perturbations.

2.4. The UV safety of M-flation

The spectrum of spectator modes of M-flation is hierarchical and the masses vary from $\sim \sqrt{\lambda_{\rm eff}}\phi$ to $\sim N\sqrt{\lambda_{\rm eff}}\phi$. For our inflationary trajectories (*cf.* next section) this is $10^{-6}M_{\rm pl}$ to $10^{-1}M_{\rm pl}$. In order for a spectator to contribute to the reduction of the UV cutoff Λ, from $M_{\rm pl}$, its mass has to be lighter than the cutoff. Since the masses of the spectators have a hierarchical structure and most of them are around the upper bound $10^{-1}M_{\rm pl}$, only about $N_S \simeq 10^5$ of them contribute to lowering the cutoff (If the large mass spectators are included, the cutoff will become smaller than the mass of contributing spectators). Therefore, the effective UV cutoff in M-flation model is very close to $5\times 10^{-3}M_{\rm pl}$ [21,16]. See Section 4 for more discussions on this point.

3. Predictions of M-flation in the (n_S–r) plane

As noted in [19,20], the inflationary potential for the supersymmetric choice (3.1), which reduces to the following condition in terms of effective couplings

$$\kappa_{\rm eff}^2 = \frac{9}{8}\lambda_{\rm eff}m^2, \tag{3.1}$$

the potential takes a double-well form

$$V(\phi) = \frac{\lambda_{\rm eff}}{4}\phi^2(\phi-\mu)^2 \tag{3.2}$$

where

$$\mu = \sqrt{\frac{2}{\lambda_{\rm eff}}}m. \tag{3.3}$$

The form of the potential is a Higgs-like double-well potential, Fig. 1, where, as we will notice shortly, μ has to take super-Planckian values to be able realize inflation. Such super-Planckian values for the effective inflaton could be easily accommodated within the setup of M-flation, as the physical excursion are normalized by $\sqrt{N(N^2-1)/4}$ from the roaming of the effective inflaton (2.10). In the stringy picture, under the influence of the RR six-form flux, two of the dimensions perpendicular to the D3-branes are blown up to a fuzzy sphere, S^2, and we have a polarized fuzzy D5 brane. The $SU(2)$ direction will play the role of the radius of the fuzzy sphere which is our effective inflaton field ϕ. Depending on the initial radius of this giant D5 brane, the radius of S^2 can either increase or shrink. We will confine ourselves to region $\phi \geq 0$ in the rest of this paper. We analyze each of these regions separately.

3.1. Region (a)

In this region, the initial condition for the effective radius of the giant D5 brane is such that it shrinks until it is stabilized at the $\phi = \mu$ vacuum. The total number of e-folds, N_e is

$$M_P^2 N_e = \int_{\phi_f}^{\phi_i} \frac{d\phi V}{V'} = \frac{1}{8}(y-x) - \frac{\mu^2}{32}\ln\left(\frac{\mu^2+4y}{\mu^2+4x}\right), \tag{3.4}$$

where ϕ_f and ϕ_i are respectively the values of the field at the end and N_e e-folds before the end of inflation. For convenience we have defined

$$x \equiv \phi_f(\phi_f-\mu), \qquad y \equiv \phi_i(\phi_i-\mu). \tag{3.5}$$

The end of inflation is defined as the point where the first slow-roll parameter,

$$\epsilon = \frac{1}{2}M_P^2\left(\frac{V'}{V}\right)^2, \tag{3.6}$$

becomes equal to one which yields ϕ_f such that

$$x = 4M_P^2 + M_P\sqrt{16M_P^2 + 2M_P^2\mu^2}. \tag{3.7}$$

The scalar spectral index at ϕ_i is

$$n_S - 1 = 2\eta - 6\epsilon \tag{3.8}$$

at ϕ_i which can be used to eliminate y in favor of n_S and μ/M_P

$$\frac{y}{M_P^2} = \frac{12+\sqrt{144+8(1-n_S)\frac{\mu^2}{M_P^2}}}{1-n_S}. \tag{3.9}$$

Inserting these values for x and y in (3.4), we find an equation for μ/M_P for a given n_S. We tried to solve this equation numerically with $N_e = 60$ and n_S between the 2σ interval of n_S allowed by Planck,

$$0.9457 \leq n_S \leq 0.9749. \tag{3.10}$$

However not for every value of n_S in this interval (3.10), Eq. (3.9) yields a real solution for μ. The smallest n_S, the model can achieve is the one of $\lambda\phi^4/4$ model,

$$n_S^{\rm min} = \frac{58}{61} \simeq 0.9508, \tag{3.11}$$

which could be obtained in the limit of $\mu \to 0$ of potential (3.2). It also turns out that for $n_S > n_{S,60}^{\rm max}$, Eq. (3.4) does not have a real solution for μ. As one can see in the left plot of Fig. 2, for $n_{S,60}^{\rm max} \simeq 0.9670$, μ/M_p goes to infinity. Thus values of $n_S > n_{S,60}^{\rm max}$ cannot be obtained in this branch of the potential for $N_e = 60$. To recap, for $N_e = 60$

$$0.9508 \lesssim n_S^{60} \lesssim 0.9670. \tag{3.12}$$

At the end, the value of $\lambda_{\rm eff}$ could be obtained, matching the amplitude of density perturbations,

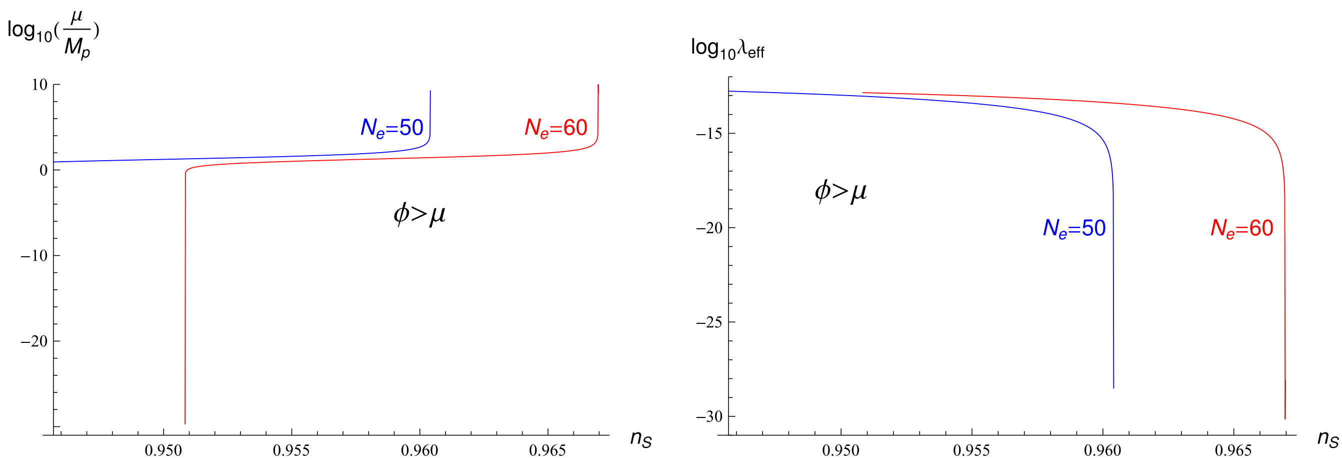

Fig. 2. Left and right plots respectively demonstrate μ vs. n_S and $\lambda_{\rm eff}$ vs. n_S in the inflationary region where $\phi > \mu$. Blue and red curves are respectively for $N_e = 50$ and $N_e = 60$. As $\mu \to 0$, the predictions of the model approaches the $\lambda\phi^4$ model. As $\mu \to \infty$ n_S reaches its maximal value, the scalar spectral index of the quadratic model. For $N_e = 50$ and $N_e = 60$, the maximum n_S's are respectively $n_S^{50} \lesssim 0.9604$ and $n_S^{60} \lesssim 0.9670$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

$$A_S = \frac{V(\phi_i)}{24\pi^2 M_{\rm pl}^4 \epsilon(\phi_i)}, \tag{3.13}$$

with the observed value by Planck, $A_S^{\rm Planck} \simeq 2.1955 \times 10^{-9}$. In the right plot of Fig. 2, one can see how $\lambda_{\rm eff}$ is related to n_S. For large values of μ/M_P when $n_s \to 0.9670$, the potential becomes shallower by decreasing the value of $\lambda_{\rm eff}$, *i.e.* $\lambda_{\rm eff} \to 0$. In this limit the potential approaches the $m^2\phi^2/2$ potential with mass $m = \mu\sqrt{\lambda_{\rm eff}/2}$. The largest value $\lambda_{\rm eff}$ obtains is for the quartic potential, $\mu = 0$,

$$\lambda_{\rm eff}^{60} \lesssim 1.4320 \times 10^{-13}. \tag{3.14}$$

In the allowed range of n_S for $N_e = 60$, within 2σ range of the Planck data, the tensor-to-scalar ratio varies in the range

$$0.132 \lesssim r_{60} \lesssim 0.262, \tag{3.15}$$

see the left plot in Fig. 3. It is also interesting to obtain the amount of excursion of the effective inflation for $N_e = 60$, $\Delta\phi_{60}$ which turns out to be constrained as

$$14.15M_{\rm pl} \lesssim \Delta\phi_{60} \lesssim 19.26M_{\rm pl}. \tag{3.16}$$

Smaller values of $\Delta\phi_{60}$ is obtained for larger values of n_S and μ, see the right plot in Fig. 3. Even though $\mu \to \infty$ as $n_S \to 0.9670$, the displacement of the field decreases and tends to 14.15 $M_{\rm pl}$. In fact the larger the μ, the smaller the amount of the effective inflaton field displacement.

It is expected that in future the n_S is measured with $\sigma(n_S) = 0.0029$ [23] with the CMBPOL experiment. Assuming the central value for n_S remains at its corresponding value from the Planck experiment, $n_S = 0.9603$, r still can vary in the range, $r \in [0.198, 0.22]$ in the 1σ range around the central value.

If $N_e = 50$, maximum n_S that could be reached is $n_{S,50}^{\rm max}$, turns out to be $\simeq 0.9604$. For larger values of n_S, the equation for μ does not have any real solution. The lowest value which could be obtained for $N_e = 50$ in this potential is again for the $\lambda\phi^4$ theory ($\mu = 0$) which is 16/17. However this value for n_S is already outside the 2σ region for n_S, see (3.10). Thus we assume that

$$0.9457 \leq n_S \lesssim 0.9604. \tag{3.17}$$

In this range of n_S, the quartic coupling and μ respectively vary such that

$$\lambda_{\rm eff}^{50} \leq 9.2945 \times 10^{-14}, \qquad \mu_{50} \gtrsim 8.64\ M_{\rm pl}. \tag{3.18}$$

The tensor-to-scalar ratio r_{50} and the effective inflaton displacement $\Delta\phi_{50}$ vary as

$$0.158 \lesssim r_{50} \leq 0.251, \qquad 12.8M_{\rm pl} \lesssim \Delta\phi_{50} \lesssim 15M_{\rm pl} \tag{3.19}$$

The smallest value of $\Delta\phi_{50}$ again is obtained when $\mu \to \infty$. If one allows for the variation of n_S in a one sigma interval then r_{50} varies in [0.158, 0.234]. If future experiments manage to confine n_S to $\sigma(n_S) = 0.0029$, assuming the central value is not changed from the Planck experiment, $r \in [0.158, 0.195]$.

In the inflationary region, $\phi > \mu$, the mode with the largest isocurvature amplitude is $j = 0$ gauge mode which is massless. Numerical computation of the amplitude of this isocurvature mode suggests that, for $N_e = 60$, $P_{iso}^{A,0}/P_S \lesssim 1.64 \times 10^{-2}$, which is obtained for the $\mu = 0$ case. To obtain this we have solved the equation for isocurvature perturbations assuming the mass expression for each spectator mode. More details of the analysis may be found in [19]. Our analysis also suggests that with increasing μ towards infinity, the $P_{iso}^{A,0}/P_S$ goes to 8.27×10^{-3} for $N_e = 60$. For $n_S = 0.9603$, $P_{iso}^{A,0}/P_S$ is about 1.24×10^{-2} at the horizon scale. If one identifies the massless $U(1)$ mode of the gauge field with the electromagnetic field of the Standard Model, one has a natural explanation for the observed cosmological magnetic fields [25] during inflation. In this approach the $U(1)$ Electromagnetic field is enhanced as an isocurvature mode and has an amplitude smaller but still comparable to the scalar perturbations. The next isocurvature mode with the largest amplitude is $j = 1$ β-mode which has a constant mass equal to the mass of the inflaton, m^2. As n_S varies in the legitimate range, $P_{iso}^{\beta,1}/P_S \lesssim 1.64 \times 10^{-2}$. The upper bound is again obtained for $\lambda\phi^4$ theory in which $m = \mu = 0$. However the ratio $P_{iso}^{\beta,1}/P_S$ goes to $\simeq 3.51 \times 10^{-5}$, as $\mu \to \infty$. For a given, n_S, the $j = 1$ β-mode goes is smaller in comparison with the $j = 0$ gauge mode, as it is massive during inflation. For $n_S = 0.9603$, $P_{iso}^{\beta,1}/P_S \simeq 4.33 \times 10^{-4}$.

3.2. Regions (b) and (c)

These two regions correspond to the kind of inflationary model which is known as hilltop models. One can follow the same procedure to obtain the relevant parameters for different values of n_S. The predictions of these two regions in the (n_S, r) plane are the same due to the symmetry of the potential around $\phi = \mu/2$. Therefore, we will focus on the case where $\frac{\mu}{2} < \phi_i < \mu$. The only differences with the computations of the previous section are that

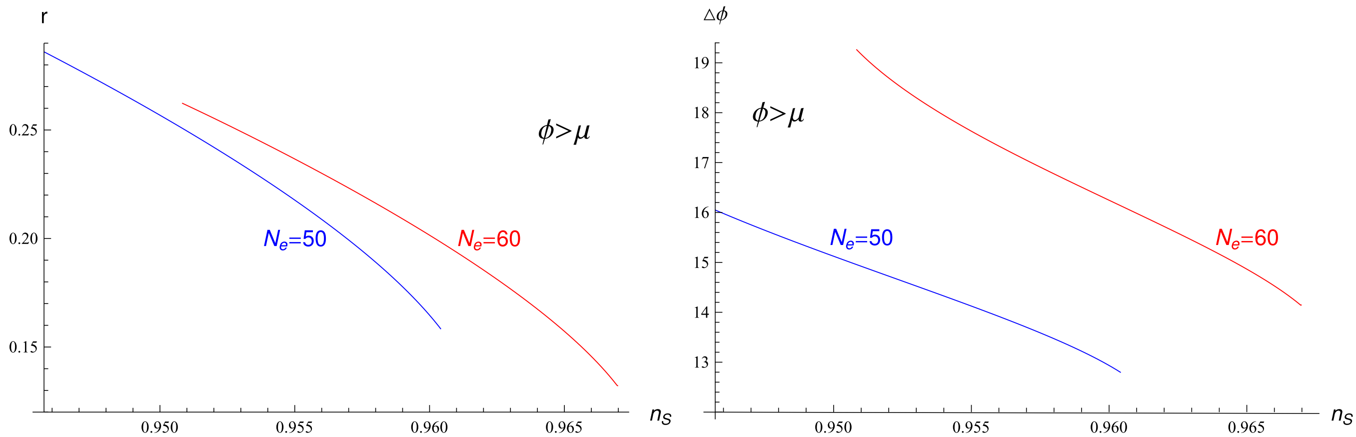

Fig. 3. Tensor-to-scalar ratio r and field roaming $\Delta\phi$ vs. n_S as n_S changes within the allowed ranges for $N_e = 50$ and $N_e = 60$ when inflation happens in the $\phi > \mu$ region.

$$x = 4M_{\rm pl}^2 - M_{\rm pl}\sqrt{2\mu^2 + 16M_{\rm pl}{}^2} \tag{3.20}$$

and

$$y = \frac{12 - \sqrt{144 + 8(1 - n_S)\frac{\mu^2}{M_{\rm pl}^2}}}{1 - n_S}, \tag{3.21}$$

but the rest of the computations could be applied.

For the central value of n_S from the Planck data, $n_S = 0.9603$, for $N_e = 60$, the prediction of the model for r is 0.038. The symmetry breaking vacuum and the quartic coupling are respectively, $\mu = 33.78M_{\rm pl}$ and $\lambda_{\rm eff} = 7.0306 \times 10^{-14}$. Again for $N_e = 60$, one finds out that within the 2σ range of n_S allowed by the Planck experiment, the equation for μ does not have any real solution for $n_{S,60}^{\rm max} \gtrsim 0.967$, again the scalar spectral index of the quadratic inflationary model. In fact this value for scalar spectral index is obtained in the limit that $\mu \to \infty$. Thus we assume

$$0.9457 < n_S^{60} \lesssim 0.9670 \tag{3.22}$$

As can be seen from the Fig. 4, $\lambda_{\rm eff}$ and μ_{60} in this range of n_S varies as

$$\lambda_{\rm eff}^{60} \lesssim 7.9948 \times 10^{-14}, \qquad 25.43M_{\rm pl} \lesssim \mu_{60}. \tag{3.23}$$

The tensor-to-scalar ratio r_{60} and the field displacement $\Delta\phi_{60}$ vary in the following ranges,

$$0.016 \lesssim r_{60} \lesssim 0.132, \qquad 25.43M_{\rm pl} \lesssim \Delta\phi_{60}, \tag{3.24}$$

please see Fig. 5. If $\sigma(n_S)$ could be lowered to 0.0029, as [23] suggests, assuming the central value for n_S remains as in the Planck experiment, r varies in $[0.031, 0.048]$.

If $N_e = 50$, the maximum n_S that could be obtained in this region is $n_{S,50}^{\rm max} \simeq 0.9618$. For larger values of n_S, the equation for μ does not have any real solution and again in this limit μ goes to infinity. Varying n_S^{50} between 0.9457, the lower bound of the Planck experiment 2σ region, and $n_{S,50}^{\rm max}$, *cf.* Fig. 4,

$$\lambda_{\rm eff}^{50} \lesssim 1.8678 \times 10^{-14}, \qquad 26.78M_{\rm pl} \lesssim \mu_{50}. \tag{3.25}$$

The tensor-to-scalar ratio r_{50} and the field displacement $\Delta\phi_{50}$ vary in the following ranges, (*cf.* Fig. 5)

$$0.031 \lesssim r_{50} \lesssim 0.118, \qquad 9.36M_{\rm pl} \lesssim \Delta\phi_{50} \lesssim 11.89M_{\rm pl}. \tag{3.26}$$

If $\sigma(n_S)$ is lowered to 0.0029 still the predictions of the model for r varies between 0.064 and 0.118.

Even though the predictions of regions (b) and (c) are the same in the (n_S, r) plane, their forecast for the amplitude of isocurvature perturbations at the end of inflation are in general different, as the masses of the spectator modes are in general ϕ-dependent. The lightest mode (b) is again the $j = 0$ gauge mode. For 60 e-folds, increasing n_S from 0.9457 to its maximum 0.9670, $P_{iso}^{A,0}/P_S$ increases approximately from 9.83×10^{-4} to 6.04×10^{-3}. The next isocurvature mode with the largest amplitude in the tower is $j = 1$ β mode whose amplitude decreases from 7.67×10^{-6} to 5.96×10^{-9} as n_S increases in the allowed range.

In region (c), the mode with the largest amplitude for isocurvature perturbations is $j = 0$ α-mode. Its amplitude $P_{iso}^{\alpha,1}/P_S$ varies between 1.64×10^{-2} and 8.27×10^{-3}. The next ones are $j = 0$ gauge and $j = 1$ β-modes which have the same amplitude as region (b) due to the symmetry of their mass with respect to the symmetry $\phi \to \mu - \phi$. In region (c), the model is equipped with an embedded preheating mechanism that uses the isocurvature fields as preheat modes. The couplings of the preheat modes to the inflaton are related to the inflaton's self-couplings hence known. In [26], we numerically computed the amplitude of gravitational waves which is generated during preheating. We found that the spectrum peaks in the GHz band with $\Omega_{\rm GW}h_0^2 \propto 10^{-16}$.

We should also note that, as was discussed in [16], in the region (c), the first $\simeq 93$ modes become tachyonic, at least for a while, during the 60 e-folds of inflation. However the mass squared of these modes, despite of being tachyonic, remain of order $m^2 \simeq -0.01H^2$, where H is the inflationary Hubble parameter during inflation. The exact trajectory may be quite complicated but this suggests that even though the inflaton may roll off the $SU(2)$ direction along those unstable ones, it may again lead to a hilltop-like inflation along those orthogonal directions. The results that we have quoted above for the isocurvature perturbations are obtained neglecting backreaction of quantum fluctuations of those tachyonic directions.

4. Concluding remarks

M-flation is a many-field inflationary model that could be realized within string theory from the dynamics of a stack of D-branes exposed to the six-dimensional flux configuration. Due to the Myers effect, two of the dimensions perpendicular to the D3 branes puffs up to a fuzzy sphere whose radius plays the role of inflaton. Requiring the 10-dimensional background to be a solution to the supergravity equations, forces the shape of the potential to be a symmetry-breaking double-well potential. In the introduction we summarized some specific theoretical features of M-flation and its theoretically appealing features. Here we point out two such other features:

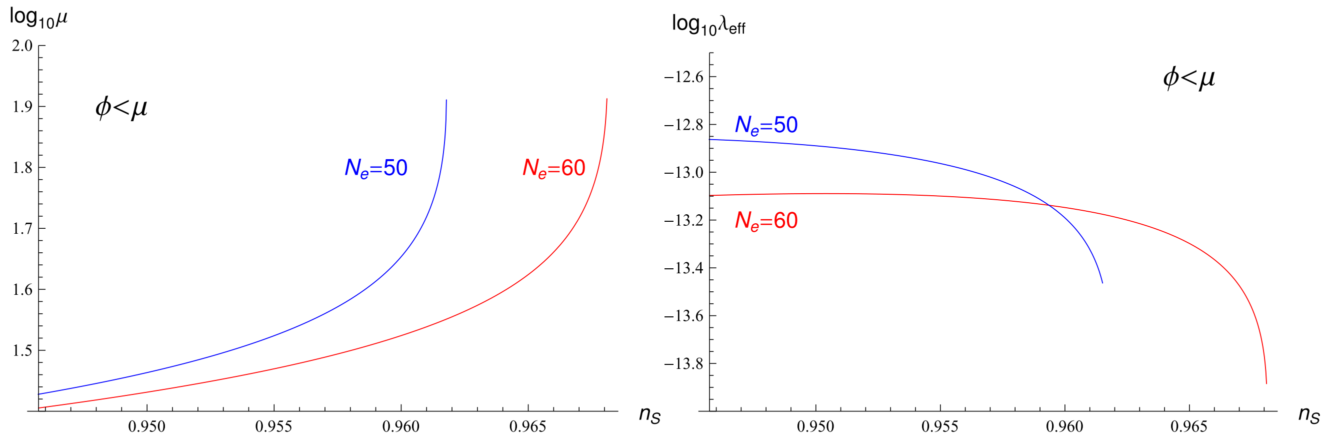

Fig. 4. μ and $\lambda_{\rm eff}$ vs. n_S in the region $\phi < \mu$.

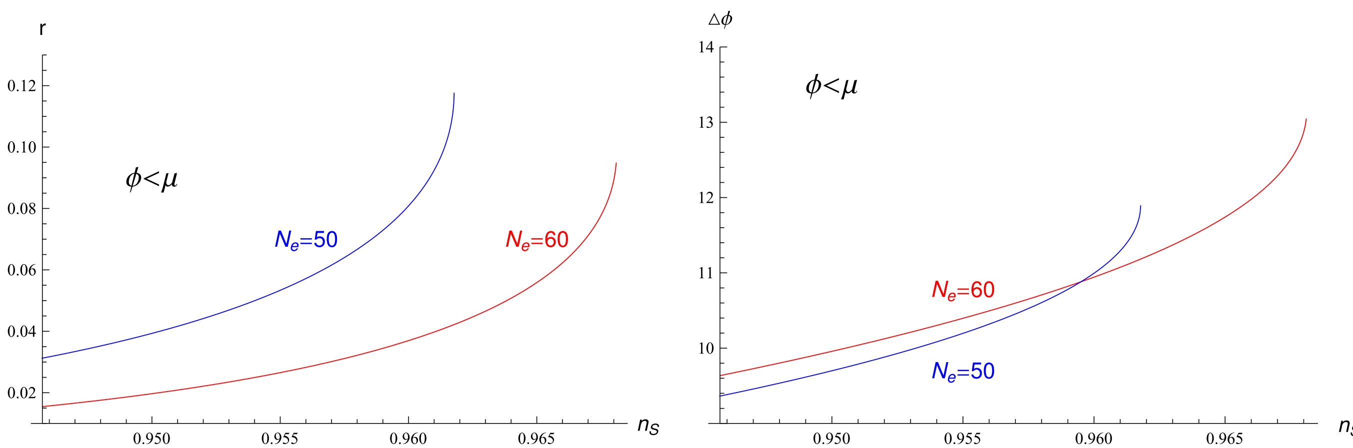

Fig. 5. r and $\Delta\phi$ vs. n_S in the region $\phi < \mu$.

(1) *Sub-Planckain field displacements*. We note that the individual physical fields displacements during our M-flation is $10^{-6}M_p \ll M_p$. Such displacements can even be produced by sub-Planckian thermal pre-inflation fluctuations which "collectively" give rise to an "effective" super-Planckian displacement needed for the "effective" inflaton. This is different from usual chaotic models in which the field should take super-Planckian v.e.v.'s which could only be achieved anthropically or if the thermal bath before inflation has (super-)Planckian energy densities.

(2) *Stability of inflationary trajectories*. Given large number of spectator modes one may worry about their backreaction destroying the slow-roll trajectories. One can distinguish two such effects, one is coming from the isocurvature modes which have crossed the horizon and became classical and hence have a non-zero v.e.v. which may backreact on inflationary trajectory; or consider the effects of sub-horizon quantum modes and the deformation of the potential due to quantum loop effects. The latter was discussed in detail in [19] and discussed that basically due to the shape of potential (induced from supersymmetric 10d background) such quantum corrections are small. The former, was also discussed in [16,19], and here we give a short review of the discussion: The power spectrum of the spectators (which is basically a measure of the energy carried by these modes and hence a measure of their backreaction) drops down exponentially with their mass as $\exp(-m/H)$. On the other hand, the mass of states grows with quantum number j, which also controls their degeneracy $2j+1$. Therefore, the growth of degeneracy is basically overshadowed by the exponential suppression of the modes due to their mass and in the end the effects of these isocurvature modes remain small. (Recall that as we already discussed the power spectrum of lightest isocurvature modes is of order 10^{-2} less than the power spectrum of curvature perturbations.) Altogether the total backreaction of the isocurvature modes remains small and do not undermine M-flation slow-roll trajectories.

We note that to have a full embedding or derivation of gauged M-flation from string theory we need to also discuss how to get four-dimensional gravity. For the latter we need to construct CY_3 compactification of 10d string theory to four dimensions and make sure about the consistency of the compactification when we probe the CY_3 with a stack of N D3-branes. We hope to address this latter in our upcoming publications.

We also discussed phenomenological appealing features of gauged M-flation with double-well potential. In the region $\phi > \mu$ predictions of the model is at the central point of the BICEP2 data, $r = 0.2$ [2], assuming that n_S is at the central value Planck yields for the scalar spectral index, $n_S = 0.9603$. Due to the hierarchical structure of the mass spectrum of the spectator species, the model is immune to super-cutoff excursion problem, which undermines N-flation [16]. Also in the $\phi > \mu$ region, the Hessian mass matrix of all spectator modes is a positive-definite, suggesting that the $SU(2)$ sector in this direction is an attractor. $SU(2)$ sector is also an attractor in the hilltop inflationary region $\mu/2 < \phi < \mu$. Since the inflationary potential can in principle approach the Planckian energy densities in the $\phi > \mu$ region, the energy density in the inflationary Planck length patch at that epoch can have a vacuum energy comparable to the energy density of the anisotropies and thus can dilute them. Thus it is more likely to lead to inflation, which justifies the large value of r observed by the BICEP2 region.

In the hilltop region $0 < \phi < \mu/2$ the first $\simeq 93$ modes become tachyonic, at least for a while, during the 60 e-folds of inflation [16]. However the mass squared of these modes, despite of being tachyonic, remain of order $m^2 \simeq -0.01H^2$, where H is the inflationary Hubble parameter during inflation. This suggests that even though the inflaton may roll off the $SU(2)$ direction along those unstable ones, it may lead to a hilltop-like inflation along those orthogonal direction. We postpone the thorough analysis of this point to future works.

In this note, we analyzed the predictions of M-flation in different inflationary regions assuming that n_S varies in the 2σ region allowed by the Planck data [1]. Even though the 2σ region is small, we notice that in different inflationary regions some ranges of n_S might not be approached by the model. For $N_e = 60$, values of $0.9508 \lesssim n_S \lesssim 0.9670$ could be achieved. The minimum value for n_S is obtained for $\mu = 0$, *i.e.* the quartic chaotic model. Values close to 0.9670 is obtained for $\mu \to \infty$. The field displacement still remains of order $\mathcal{O}(10)M_{\rm pl}$ despite μ becoming large. What happens is that $\lambda_{\rm eff}$ tends towards zero as μ increases. Nonetheless, if one demands to avoid the "hyper-Planckian" v.e.v.'s for μ, say $\mu \lesssim 100M_{\rm pl}$, maximum n_S will reduce to 0.9650. Another potentially observable signature of the model in the region $\phi > \mu$, is a small isocurvature perturbation mode with amplitude of $\simeq 10^{-2}$ which is obtained from $j = 0$ gauge mode. Of course whether this mode would be observable, depends on the details of the reheating.

Although BICEP2 claims to observe a B-mode signature corresponding to $r_{0.05} = 0.2^{+0.07}_{-0.05}$, the Planck data so far has only suggested an upper bound on the amplitude of tensor modes $r_{0.002} < 0.11$. There has been various proposals to patch up the BICEP2 results with the Planck upper bound on the B-modes [24]. In [28], based on our previous analysis in [29] for excited states, we proposed a scenario to reconcile the large scale models of inflation with the Planck data and BICEP data. For that, one has to assume that the scale of new physics is about $M = \text{few} \times 10H$. To reconcile the Planck and BICEP2 data, we proposed two approaches to solve the tension between the Planck and BICEP2 data: (i) creating a running scalar spectral index with running of order $dn_S/d\ln k \simeq -0.02$ [27], or (ii) creating a blue tensor spectrum for the tensor modes. Both these two approaches could be easily accustomed to M-flation assuming non-Bunch–Davis, excited initial state for cosmic perturbations.

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